University of California, Berkeley
Physics H7A Fall 1998 (*Strovink*)

EXAMINATION 2

Directions. Do all three problems (weights are indicated). This is a closed-book closed-note exam except for one $8\frac{1}{2} \times 11$ inch sheet containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he or she will not give hints and will be obliged to write your question and its answer on the board. Don't use a calculator, which you don't need – roots, circular functions, *etc.*, may be left unevaluated if you do not know them. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

1. (30 points) A cylinder of mass m and radius R initially is at rest on *frictionless* ice. About its C.M., the cylinder's moment of inertia is $mR^2/2$.
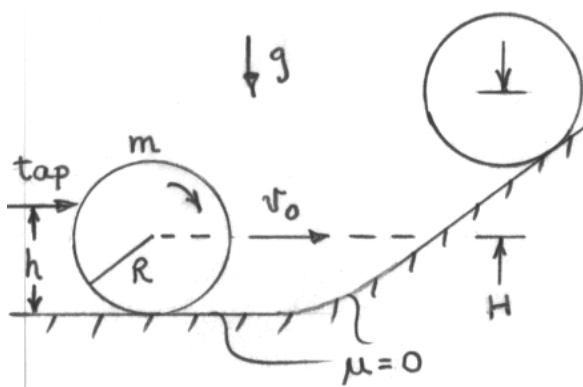

a. (15 points) The cylinder receives an instantaneous horizontal tap at a point on its circumference that is a vertical distance h above the ice. Immediately thereafter, it is observed to roll without slipping at a velocity v_0, even though the ice is frictionless. Calculate the value of h.

b. (15 points) The cylinder continues to roll without slipping as long as the frictionless ice remains flat. Eventually the ice slopes upward to form a hill as shown, all the while staying frictionless. To what maximum height H will the cylinder rise?

2. (30 points) A thin massless rod of length H hangs freely pivoted from the ceiling. At its end is a bead of mass m, negligible in size compared to H, acted upon by gravity.
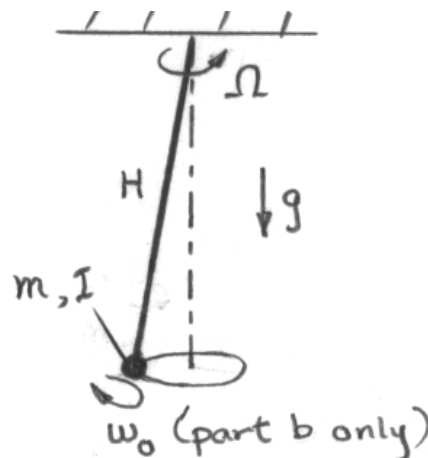
a. (15 points) For this part, consider the bead and the rod to be glued together. The C.M. of the bead is observed to travel uniformly around a horizontal circle that is centered below the rod's pivot. The circle's radius is much smaller than H but much larger than the bead. With what angular velocity Ω does the bead move on this path?

b. (15 points) For this part, the bead is no longer glued to the rod; instead it spins about the long axis of the *rod* with a constant, large angular velocity ω_0. About this axis, the moment of inertia of the bead is I. Again, the C.M. of the bead is observed to travel uniformly around a horizontal circle centered below the pivot. What restriction(s) can be placed on the radius r of this circle?

3. (40 points) A tiny pebble moves on the frictionless inner surface of a vertical cone that has a half-angle of $45°$. It is observed to be in uniform circular motion with constant velocity v_0. (You are *not* given the radius of this circle!)
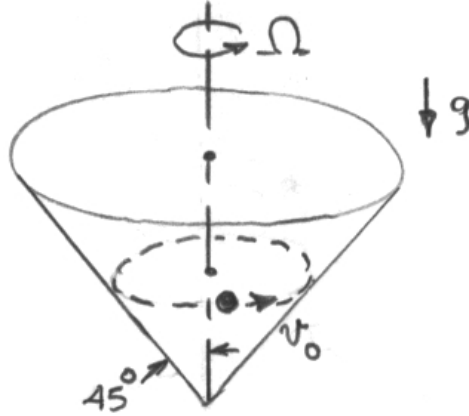
a. (15 points) what is the angular frequency Ω of this uniform circular motion?

b. (15 points) The orbiting pebble is now nudged so that its new orbit differs very slightly from the original circle. The nudge causes the orbit radius (the perpendicular distance from the pebble to the cone axis) to oscillate sinusoidally about its mean value. (All the while the pebble remains in contact with the inside surface of the cone.) Calculate the angular frequency ω_r of this small radial oscillation.

c. (10 points) Is the perturbed orbit "closed" – that is, does the perturbed orbit ever repeat itself? Explain.